Examples of 3-Dimensional Geometric Manifolds

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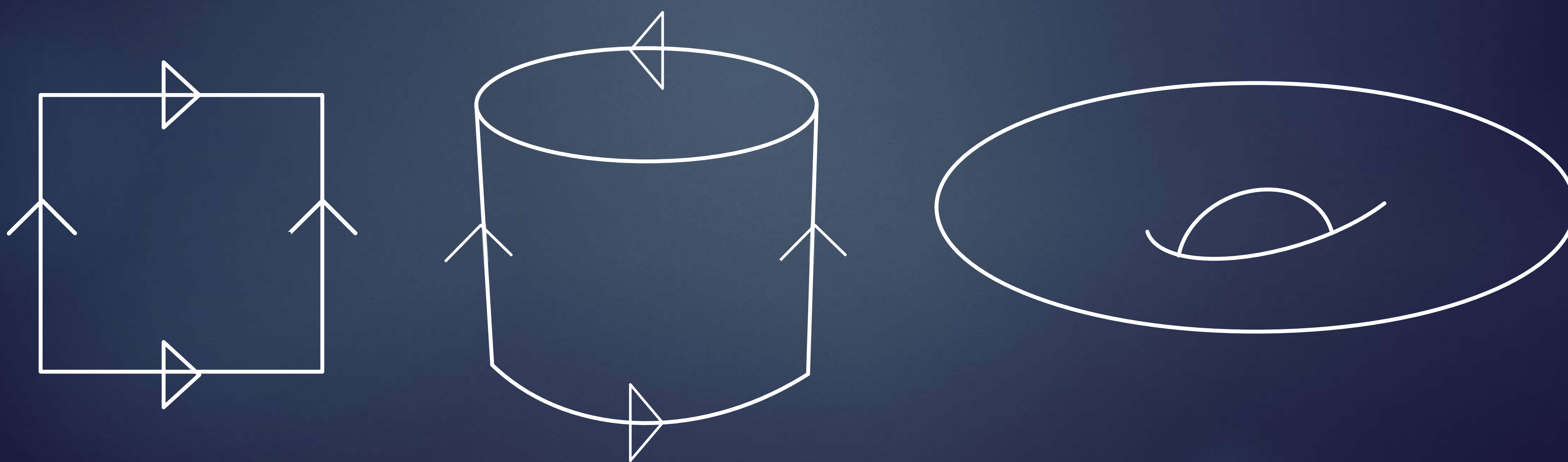
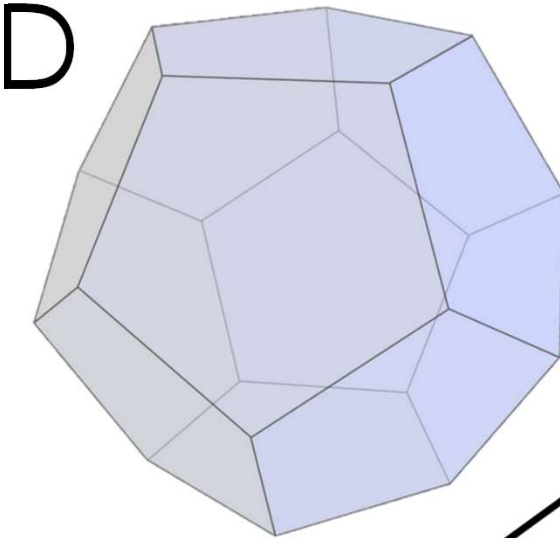
Face Pairing

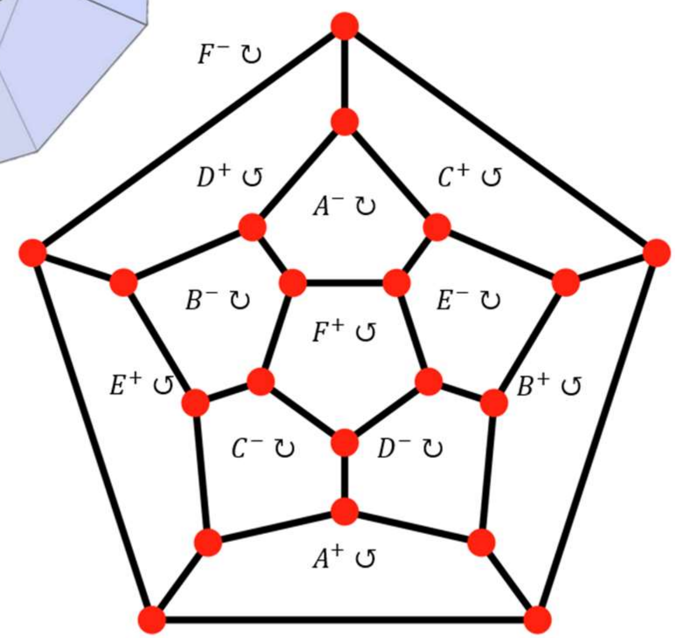
- Consider pairing opposite sides of the square.
- First the square becomes a cylinder; the next pairing creates a torus.
- Note the torus is called a 2-dimensional manifold.

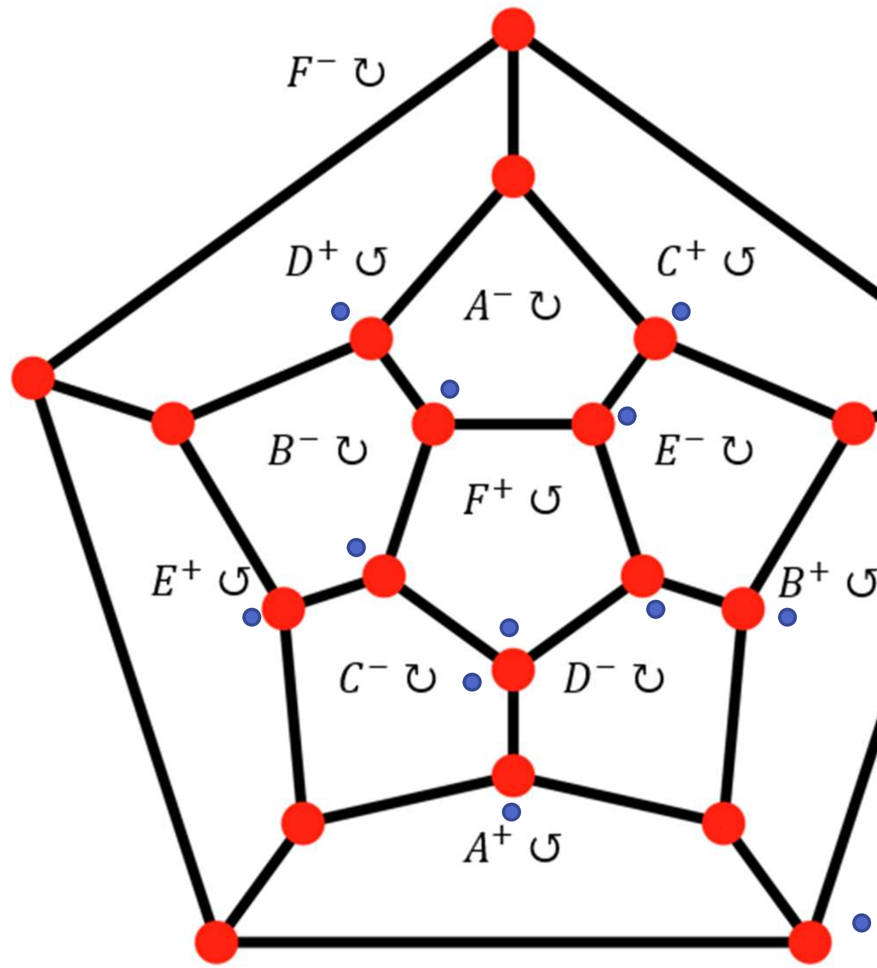


Face Pairing 3D



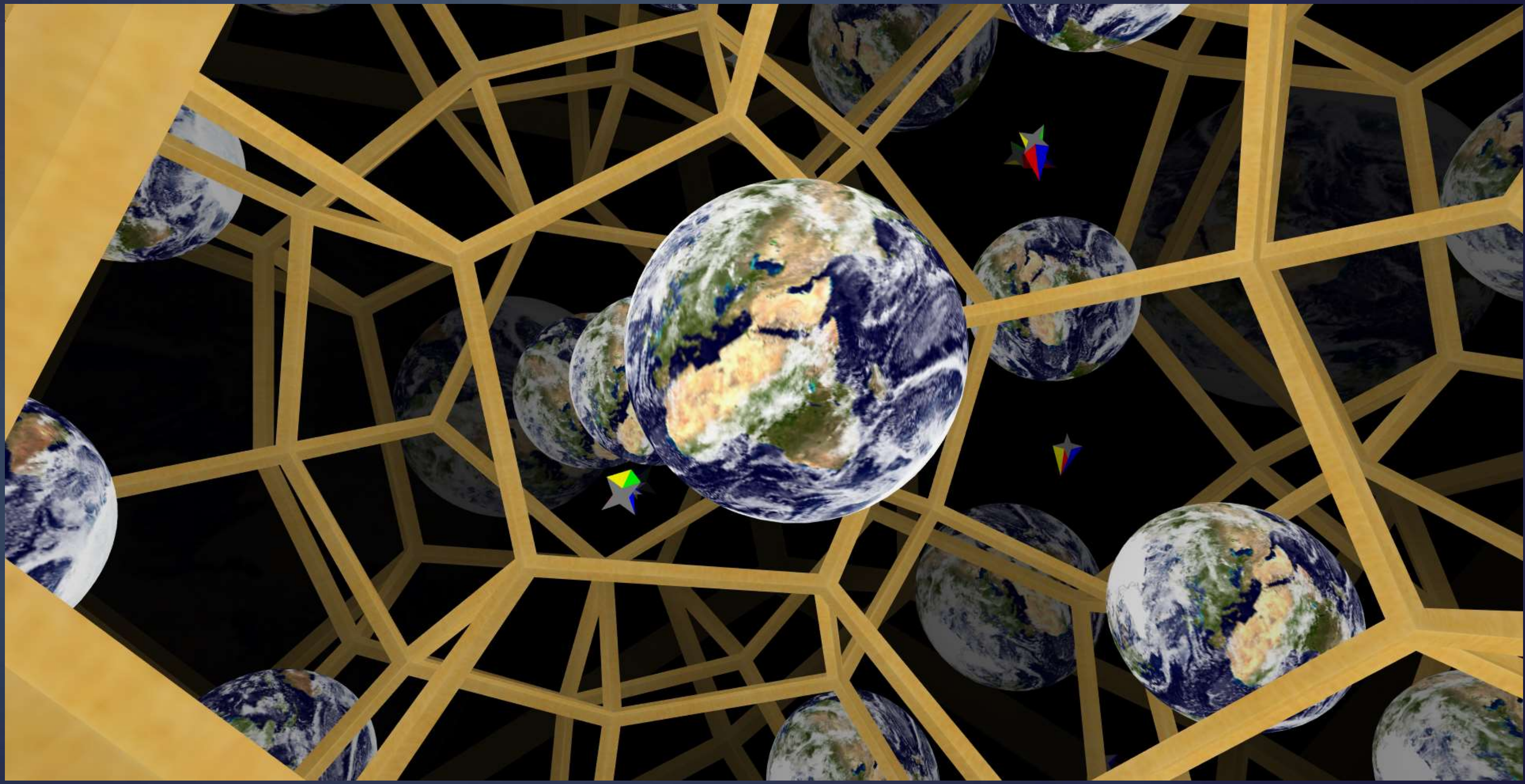
- Now consider a 3-dimensional dodecahedron (a solid shape with 12 pentagons for faces).
- Pair opposite faces; apply $\frac{1}{10}$ of a rotation in the direction of a right hand screw.





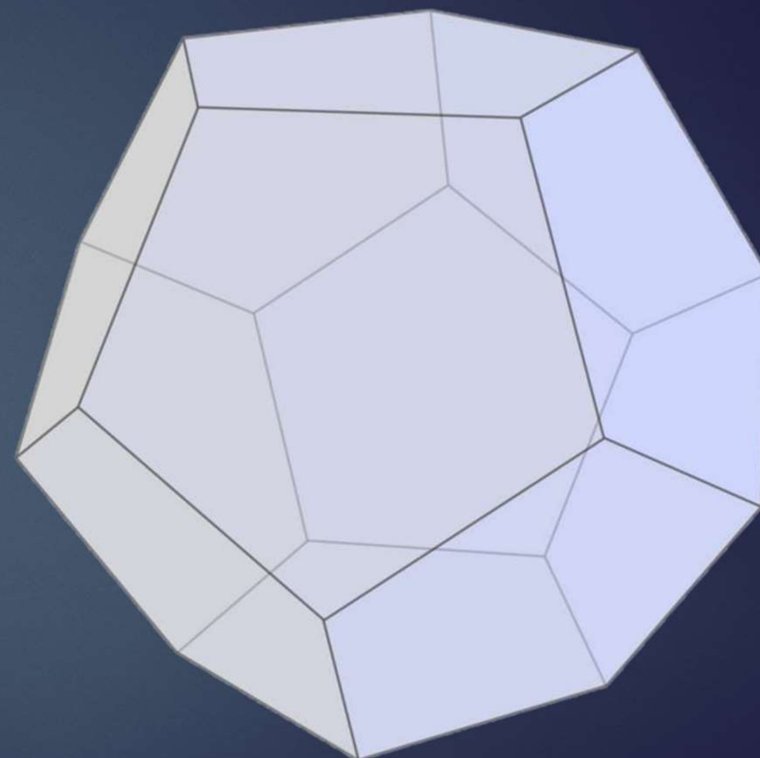
$F^- \circlearrowright$
$D^+ \circlearrowleft$
$C^+ \circlearrowleft$
$A^- \circlearrowright$
$B^- \circlearrowright$
$E^- \circlearrowright$
$F^+ \circlearrowleft$
$E^+ \circlearrowleft$
$B^+ \circlearrowleft$
$C^- \circlearrowright$
$D^- \circlearrowright$
$A^+ \circlearrowleft$

The result is the Poincaré dodecahedral space.

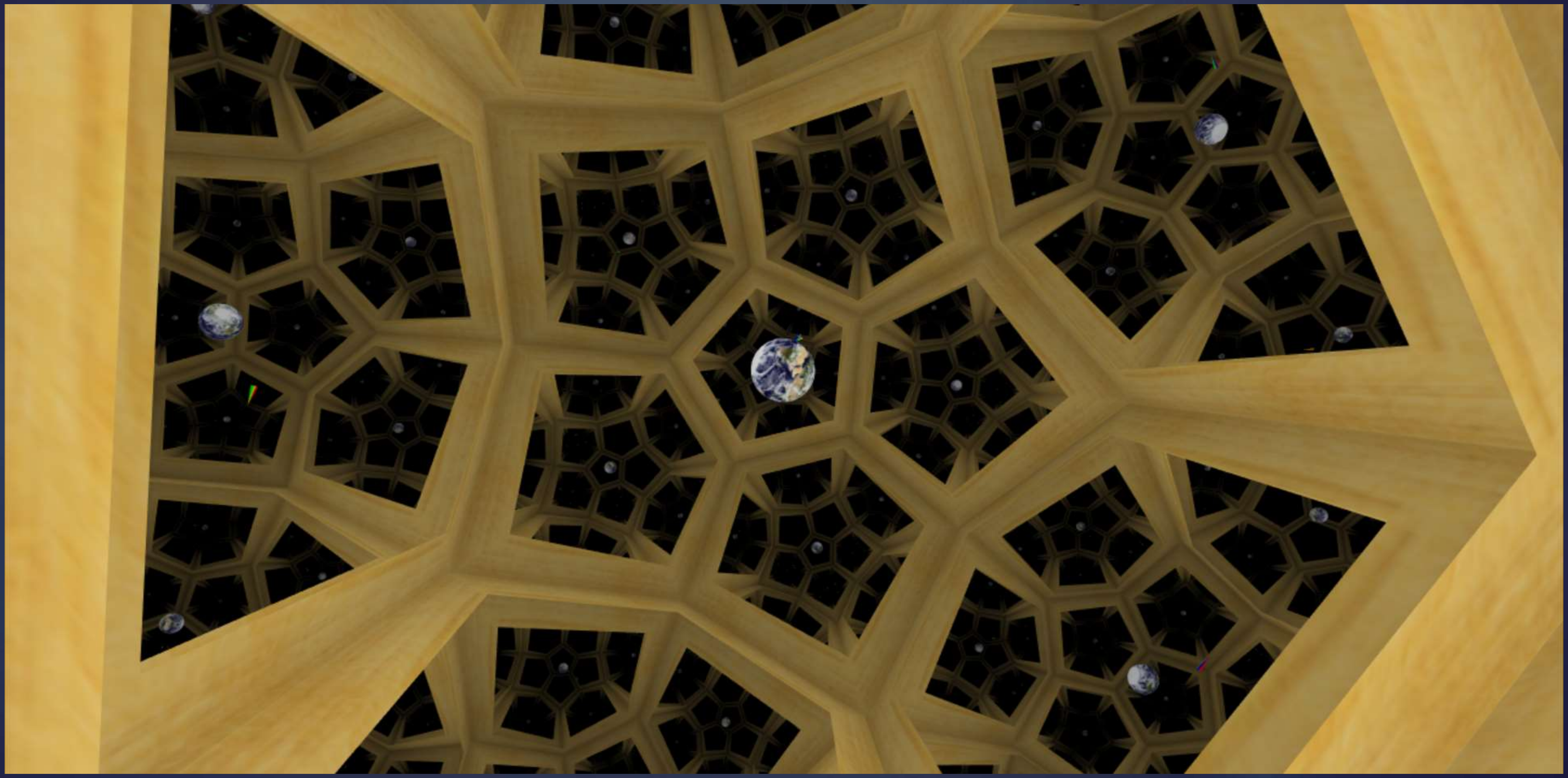


Seifert Weber Space

- Now apply a $\frac{3}{10}$ rotation to the same pairing.
- This time the edges are identified in groups of 6.

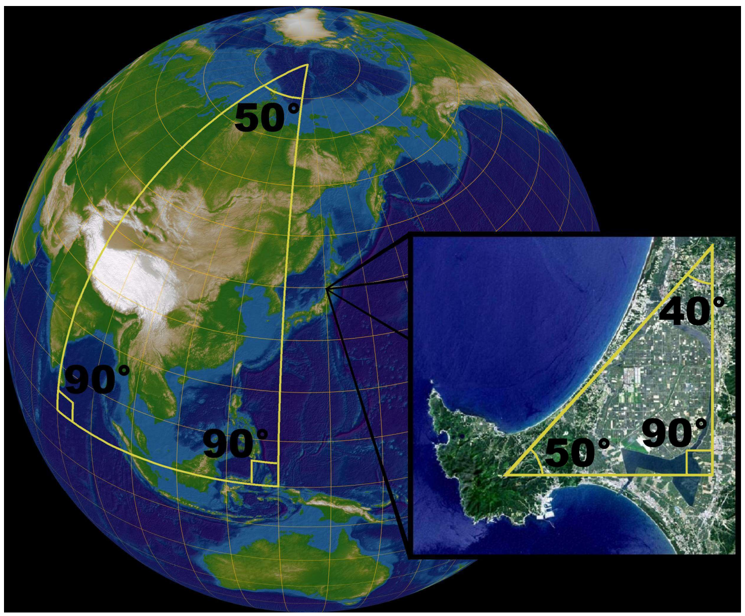


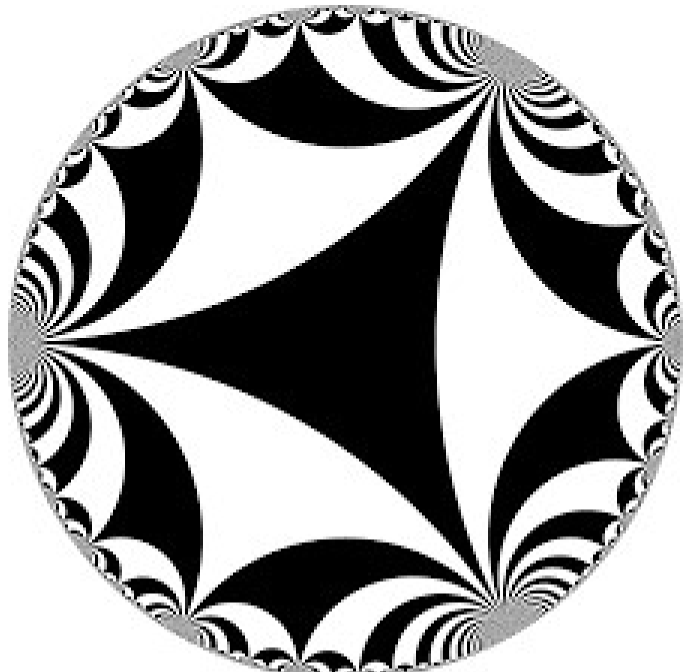
The result is the Seifert Weber space.



Spherical vs Hyperbolic Geometry

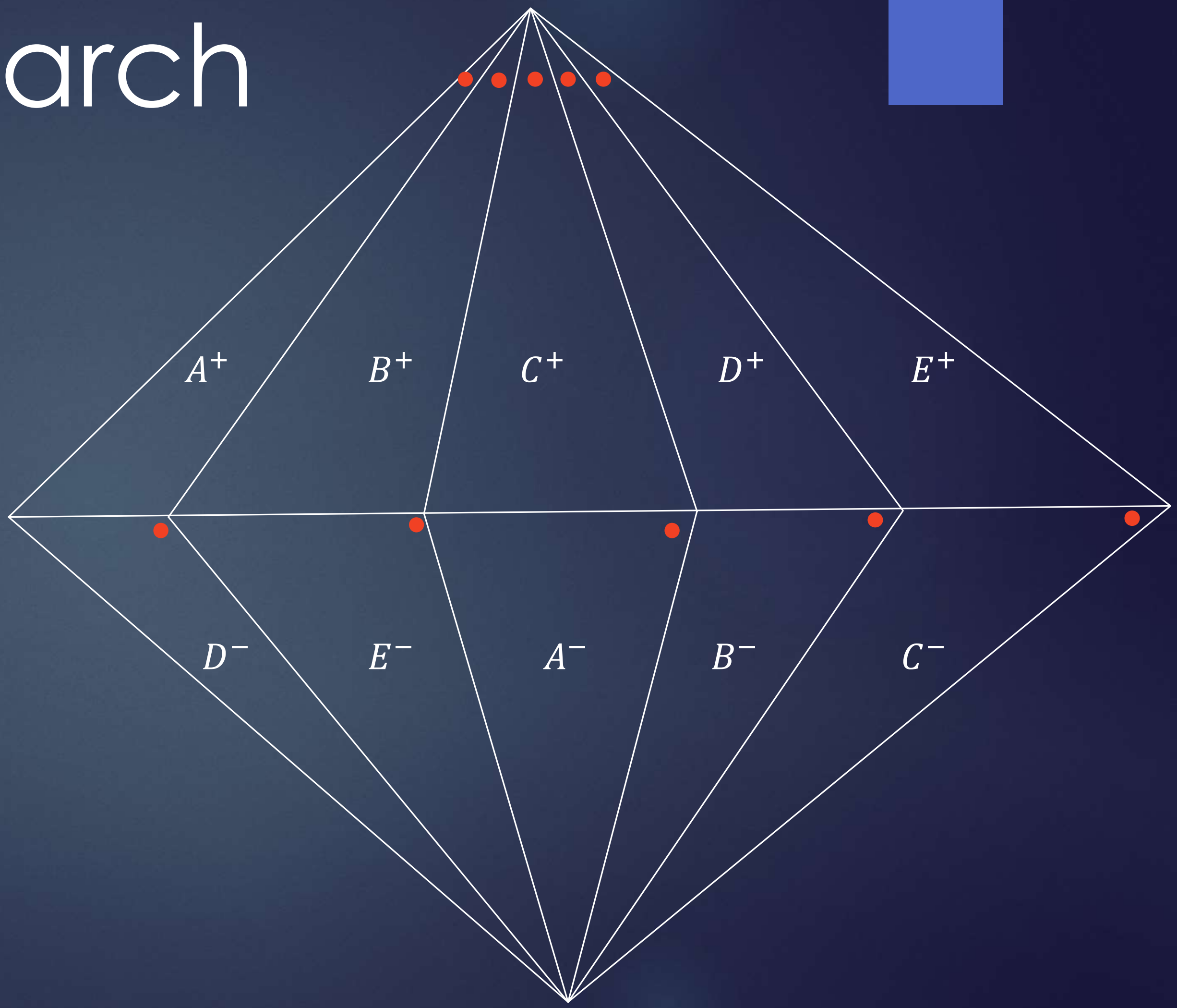
- Dihedral angle on dodecahedron is 117°.
- Poincaré: $3 \cdot 117^\circ = 351^\circ$; Seifert Weber $6 \cdot 117^\circ = 702^\circ$.
- Lower left represents 2-dimensional spherical geometry; right is hyperbolic.





Current Research

- Working to identify the geometry of the manifolds of Type Z introduced by Dr. McDermott (2022).
- Here is another model for the Poincaré dodecahedral space.



Thank You for Your Attention, Any Questions?